Republic of Namibia

MINISTRY OF EDUCATION, ARTS AND CULTURE

NAMIBIA SENIOR SECONDARY CERTIFICATE (NSSC)

MATHEMATICS SYLLABUS

ADVANCED SUBSIDIARY LEVEL

SYLLABUS CODE: 8227

GRADES 12

FOR IMPLEMENTATION IN 2021
FOR FIRST EXAMINATION IN 2021

Ministry of Education, Arts and Culture
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Mathematics Syllabus Advanced Subsidiary Level Grade 12

ISBN: 978-99945-2-247-7

Printed by NIED
Website: http://www.nied.edu.na

Publication date: December 2020

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1. INTRODUCTION

The Namibia Senior Secondary Certificate Advanced Subsidiary (NSSCAS) level syllabus is designed as a one-year course leading to examination after completion of the Namibia Senior Secondary Certificate Ordinary (NSSCO) level. The syllabus is designed to meet the requirements of the *National Curriculum for Basic Education (NCBE)* and has been approved by the National Examination, Assessment and Certification Board (NEACB).

The National Curriculum Guidelines, applicable at the stage of senior secondary education (Grades 10-12) and at equivalent stages of non-formal education, as a part of life-long learning, recognise the uniqueness of the learner and adhere to the philosophy of learner-centred education.

The Namibia National Curriculum Guidelines:

- recognise that learning involves developing values and attitudes as well as knowledge and skills;
- promote self-awareness and an understanding of the attitudes, values and beliefs of others in a multilingual and multicultural society;
- encourage respect for human rights and freedom of speech;
- provide insight and understanding of crucial "global" issues in a rapidly changing world which affects quality of life: the AIDS pandemic, global warming, environmental degradation, distribution of wealth, expanding and increasing conflicts, the technological explosion and increased connectivity;
- recognise that as information in its various forms becomes more accessible, learners need to develop higher cognitive skills of analysis, interpretation and evaluation to use information effectively;
- seek to challenge and to motivate learners to reach their full potential and to contribute positively to the environment, economy and society.

Thus the Namibia National Curriculum Guidelines should provide opportunities for developing essential skills across the various fields of study. Such skills cannot be developed in isolation and they may differ from context to context according to a field of study. The skills marked with an * are relevant to this syllabus.

The skills are:

- communication skills *
- numeracy skills *
- information skills *
- problem-solving skills *
- self-management and competitive skills *
- social and cooperative skills *
- physical skills*
- work and study skills *
- critical and creative thinking*

2. RATIONALE

Mathematics is a dynamic, living and cultural product. It is more than an accumulation of facts, skills and knowledge. The learning of mathematics involves conceptual structures, strategies of problem solving and attitudes towards and appreciation of mathematics.
Mathematical knowledge and mathematical methods of inquiry constitute an essential part of and contribute to all modern science and engineering.

Today's learners will live and work in an era dominated by computers, by worldwide communication and by a global economy. Mathematics is the key to success in this world where the economy requires workers who are prepared to absorb new ideas, to perceive patterns and to solve unconventional problems.

The senior secondary curriculum strives to prepare learners to function effectively in the 21st century by providing a basis to use mathematics in their personal and professional lives.

3. AIMS

The aims of the syllabus are the same for all learners. These are set out below and describe the educational purposes of a course in Mathematics for the NSSCAS examination. They are not listed in order of priority.

The aims are to enable students to:

1. develop their mathematical knowledge and skills in a way which encourages confidence and provides satisfaction and enjoyment;
2. develop a feel for number and measurement, carry out calculations and understand the significance of the results obtained;
3. develop an understanding of spatial concepts and relationships;
4. develop their ability to apply mathematics, in the contexts of everyday situations and of other subjects that they may be studying;
5. develop an understanding of mathematical principles;
6. develop their ability to analyse problems logically, recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve the problem;
7. use mathematics as a means of communication with emphasis on the use of clear expression;
8. develop the abilities to reason logically, to classify, to generalise and to prove;
9. experience a sufficiently wide range of mathematical topics and methods so that they can develop their appreciation of the power, elegance and structure of the subject;
10. acquire the mathematical background necessary for further study in this or related subjects.

4. ADDITIONAL INFORMATION

4.1 Guided learning hours

The NSSCAS level syllabuses are designed on the assumption that learners have about 180 guided learning hours per subject over the duration of the course (1 year), but this is for guidance only. The number of hours required to gain the qualification may vary according to local conditions and the learners' prior experience of the subject. The National Curriculum for Basic Education (NCBE) indicates that this subject will be taught for 9 periods of 40 minutes each per 7-day cycle, or 6 periods of 40 minutes each per 5-day cycle, over a year.

4.2 Recommended prior learning

It is recommended that learners who are beginning this course should have previously completed Mathematics at Namibia Senior Secondary Certificate Ordinary (NSSCO) level.

4.3 Progression

NSSCAS level Mathematics provides a suitable foundation for the study of mathematics related courses in higher education. Depending on the local university entrance requirement, it may permit or assist progression directly to university courses in Mathematics or other related course in higher education. Equally it is suitable for learners intending to pursue careers or further study in Teaching, Science, Engineering, Medicine, Statistics etc.

4.4 Grading and reporting

NSSCAS results are shown by one of the grades a, b, c, d or e indicating the standard achieved, grade a being the highest and grade e the lowest. 'Ungraded' indicates that the candidate has failed to reach the minimum standard required for a pass at NSSCAS level.

4.5 Support materials and approved textbooks

Copies of NSSCAS syllabuses, recent specimen material, question papers and examiner reports are sent to all schools. Assessment manuals in subjects, where applicable are sent to schools. Approved learning support materials are available on the Senior Secondary Textbook Catalogue for Schools. The Textbook Catalogue is available on www.nied.edu.na

5. LEARNING CONTENT

The learning content for Mathematics NSSCAS level covers the following topics

Theme 1: Mathematics 1

Topic 1: Equations, expressions, identities and inequalities
Topic 2: Sequences and series
Topic 3: Graphs and functions
Topic 4: Coordinate geometry
Topic 5: Circular measure
Topic 6: Trigonometry
Topic 7: Vectors in three dimensions
Topic 8: Differentiation
Topic 9: Integration

Theme 2: Mathematics 2

Topic 1: Algebra
Topic 2: Logarithmic and exponential functions
Topic 3: Trigonometry
Topic 4: Differentiation
Topic 5: Integration
Topic 6: Numerical solutions of equations

Theme 1: Mathematics 1

Knowledge of the content of NSSCO Mathematics is prescribed, and learners may be required to demonstrate such knowledge in answering questions.

GENERAL OBJECTIVES *Learners will:*	**SPECIFIC OBJECTIVES** *Learners should be able to:*
Topic 1: Equations, expressions, identities and inequalities	
• gain further understanding for manipulating quadratic expressions and solving quadratic equations	• complete the square for a quadratic function and use the solution to determine the turning point (vertex) of a graph of a quadratic function and sketch the graph • find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and interpret the nature and the number of real roots of the related equation $f(x) = ax^2 + bx + c$ • interpret the difference between identities and equations and use identities to determine unknown coefficients in polynomials • recognise and solve equations in x which are quadratic in some function of x e.g. $x^4 – 5x^2 + 4 = 0$ for $k = x^2$ • find the equation of a graph (quadratic and cubic) given sufficient information • solve quadratic inequalities in one unknown

GENERAL OBJECTIVES *Learners will:*	**SPECIFIC OBJECTIVES** *Learners should be able to:*
Topic 2: Sequences and series	
• gain further understanding of sequences and series	• use the expansion of $(a+b)^n$, where n is a positive integer (knowledge of the greatest term and properties of the coefficients are not required, but the notations $\binom{n}{r}$ and *n!* should be known) • recognise arithmetic and geometric progressions • use the formulae for the n^{th} term and for the sum of the first *n* terms to solve problems involving arithmetic or geometric progressions • interpret and apply the Sigma notation • use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression
Topic 3: Graphs and functions	
• understand the concept of function and use function notation	• interpret the terms function, domain, range (image set), one-one function, inverse function and composition of functions • identify the domain and the range of a given function in simple cases • determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases • form composite functions, and recognise when a given function can be expressed as a composite • sketch graphs of functions and their inverses for a specified domain

GENERAL OBJECTIVES *Learners will:*	**SPECIFIC OBJECTIVES** *Learners should be able to:*
Topic 4: Coordinate geometry	
• gain further understanding for solving problems involving coordinate geometry	• interpret the relationship between a graph and its associated algebraic equation • use the relationship between points of intersection of graphs and solutions of equations including, in simple cases, the correspondence between a line being tangent to a curve and a repeated root of an equation
Topic 5: Circular measure	
• understand the relationship between radians and degrees and solve problems involving arc length and sector area	• interpret the term radian and use the relationship between radians and degrees • use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ to solve problems concerning the arc length, sector area of a circle and segment area

GENERAL OBJECTIVES *Learners will:*	**SPECIFIC OBJECTIVES** *Learners should be able to:*
Topic 6: Trigonometry	
• understand and use trigonometric ratios and their identities	• sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians) • find the amplitude and period and sketch and interpret graphs of the form $y = a\sin(bx) + c$, $y = a\cos(bx) + c$, and $y = a\tan(bx) + c$ • use the exact values of the sine, cosine and tangent of 30°, 45°, 60°, and related angles, e.g. cos 150° = $-\frac{1}{2}\sqrt{3}$ • use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations • use the trigonometric identities $\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$ and $\sin^2\theta + \cos^2\theta \equiv 1$ to prove identities and solve trigonometric equations • find solutions, in degrees or radians and within a specified interval, of the equations $\sin(nx) = k$, $\cos(nx) = k$, $\tan(nx) = k$, and of equations easily reducible to these forms, where n is a small integer or a simple fraction (general forms of solution are not included)

GENERAL OBJECTIVES *Learners will:*	**SPECIFIC OBJECTIVES** *Learners should be able to:*
Topic 7: Vectors in three dimension	
• understand and use vectors in three dimensions	• use rectangular Cartesian coordinates to locate points in three dimensions, and use standard notations for vectors, i.e. $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, x**i** + y**j** + z**k**, $\overrightarrow{AB}$, **a** • perform addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms • use unit vectors, displacement vectors and position vectors • calculate the magnitude of a vector and the scalar product of two vectors • use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors
Topic 8: Differentiation	
• know how to find a derivative of a function and use derivatives to solve problems	• interpret the idea of the gradient of a curve, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$ $\frac{d^2y}{dx^2}$ (the technique of differentiation from first principles is not required) • use the derivative of x^n (for any rational n), together with constant multiples, sums, differences of functions, and of composite functions using the chain rule (the knowledge of product and quotient rules will not be required) • apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change (including connected rates of change) to solve problems involving displacement, velocity and acceleration • locate stationary points, and distinguish (by any method) between maximum and minimum points and points of inflexion

GENERAL OBJECTIVES *Learners will:*	**SPECIFIC OBJECTIVES** *Learners should be able to:*
Topic 9: Integration	
• know how to integrate derivatives to find their functions and use integration to solve geometric problems • know how to use integration to determine areas and volumes and solve problems involving easy kinematics	• interpret integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except –1), together with constant multiples, sums and differences • solve problems involving the evaluation of a constant of integration, e.g. to find the equation of the curve through (1, –2) for which $\frac{dy}{dx} = 2x + 1$ • evaluate definite integrals (including simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}} dx$ and $\int_1^\infty x^{-2} dx$) • use definite integration to find: - the area of a region bounded by a curve and a line, or between two curves - a volume of revolution about one of the axes - displacement, velocity and acceleration

Theme 2: Mathematics 2

Knowledge of the content of NSSCO Mathematics is prescribed, and learners may be required to demonstrate such knowledge in answering questions.

GENERAL OBJECTIVES *Learners will:*	**SPECIFIC OBJECTIVES** *Learners should be able to:*
Topic 1: Algebra	
• gain further understanding for manipulating algebraic expressions and solving absolute value equations and inequalities	• interpret and use the notation $\lvert x \rvert$, and use the relations such as $\lvert a \rvert = \lvert b \rvert = \Leftrightarrow a^2 = b^2$ and $\lvert x - a \rvert < b \Leftrightarrow a - b < x < a + b$ in solving equations and inequalities • interpret and sketch the graphs of the functions $y = \lvert ax \rvert$, $y = \lvert ax + b \rvert$ and $y = \lvert x \rvert + c$ where *a*, *b* and *c* are integers • divide a polynomial of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero) • apply the remainder and the factor theorems to polynomials and use the factor theorem to solve polynomial equations or evaluate unknown coefficients
Topic 2: Logarithmic and exponential functions	
• understand the connection between logarithm and indices and solve problems involving logarithm and exponential function	• interpret the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base) • interpret the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs • use logarithms to solve equations of the form $a^x = b$, and similar inequalities • use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept

GENERAL OBJECTIVES *Learners will:*	**SPECIFIC OBJECTIVES** *Learners should be able to:*
Topic 3: Trigonometry	
• gain further understanding for the use of trigonometric ratios and their identities	• recognise the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude • use trigonometrical identities for the simplification and exact evaluation of expressions and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of: - $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\mathrm{cosec}^2\theta \equiv 1 + \cot^2\theta$, - the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$, - the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$, - the expressions $a \sin\theta + b \cos\theta$, $a \sin\theta - b \cos\theta$ and $a \cos\theta - b \sin\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$
Topic 4: Differentiation	
• acquire further understanding for finding the derivative of a function	• use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites • differentiate products and quotients • find and use the first derivative of a function which is defined parametrically or implicitly

GENERAL OBJECTIVES *Learners will:*	SPECIFIC OBJECTIVES *Learners should be able to:*
Topic 5: Integration	
• extend their knowledge for integrating derivatives	• apply further the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax + b)$, $\cos(ax + b)$ and $\sec^2(ax + b)$ (knowledge of the general method of integration by substitution is not required) • use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as $\cos^2 x$ • use the trapezium rule to estimate the value of a definite integral • use sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or under-estimate
Topic 6: Numerical solutions of equations	
• understand alternative methods for locating roots of equations and use iterative formula	• locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change • interpret the idea of a sequence of approximations which converges to a root of an equation and use its notation • interpret how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved • use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy (knowledge of the condition for convergence is not included, but candidates should understand that an iteration may fail to converge)

6. ASSESSMENT OBJECTIVES

The abilities to be assessed in the Mathematics NSSCAS examination cover a single area, technique with application. The examination tests the ability of candidates to:

1. organise, interpret and present information accurately in written, tabular, graphical and diagrammatic forms;
2. understand relevant mathematical concepts, terminology and notation;
3. perform calculations by suitable methods, including the appropriate use of an electronic calculator;
4. recall, apply and interpret mathematical knowledge in the context of everyday situations, and work to degrees of accuracy appropriate to the context;
5. recall accurately and use successfully appropriate manipulative techniques;
6. interpret, transform and make appropriate use of mathematical statements expressed in words or symbols;
7. make logical deductions from given mathematical data;
8. respond to a problem relating to a relatively unstructured situation by translating it into an appropriately structured form;
9. analyse a problem, select a suitable strategy and apply an appropriate technique to obtain its solution;
10. apply combinations of relevant mathematical skills and techniques in problem solving;
11. present mathematical work, and communicate conclusions, in a clear, concise and logical way.

The assessment objectives above can be assessed in any question in Paper 1 and 2.

7. SCHEME OF ASSESSMENT

Assessment consists of two written papers. All candidates must take both papers, and there is no choice of questions in either paper.

Papers	Weighting of papers	Marks	Time
Paper 1 (Mathematics 1)	50%	75	2 hours
Paper 2 (Mathematics 2)	50%	75	2 hours

NOTES

1. There will be no choice of questions.

2. The syllabus prescribes that candidates will be in possession of a non-programmable scientific calculator for both papers. Three significant figures will be required in answers except where otherwise stated.
3. If the degree of accuracy is not specified in the question and if the answer is not exact, the answer must be given to three significant figures. Answers in degrees must be given to one decimal place. For π either the calculator value or 3.142 must be used.
4. Candidates will be expected to be familiar with scientific notation for the expression of compound units, e.g. 5 ms^{-1} for 5 metres per second.

8. GRADE DESCRIPTIONS

The scheme of assessment is intended to encourage positive achievement by all learners. Grade descriptions are therefore provided for judgmental grades ***a***, ***c*** and ***e*** to give a general indication of the standards of achievement likely to have been shown by learners awarded particular grades. The description must be interpreted in relation to the content specified by the Mathematics syllabus but are not designed to define that content. The grade awarded will depend in practice upon the extent to which the learner has met the assessment objective overall. Shortcomings in some aspects of the assessment may be balanced by better performance in others.

It must be clearly understood that a learner who meets the demands of a particular grade, by implication also meets the demands of all lower grades, e.g. a candidate who achieves a Grade ***a***, by implication also fulfils the criteria set for Grades ***c*** and ***e***.

*At **Grade a** the learner is expected to:*

- Apply the remainder and the factor theorem to polynomials and use the factor theorem to solve polynomial equations. Interpret the difference between identities and equations and use identities to determine unknown coefficients in polynomials. Recognise and solve equations in x which are quadratic in some function of x. Find the equation of a graph (quadratic and cubic) given sufficient information. Interpret the difference between identities and equations and use identities to determine two or more unknown coefficients in polynomials. Recognise and solve equations in x which are quadratic in some function of x e.g. $x^4 – 5x^2 + 4 = 0$ for $k = x^2$.

- Apply the expansion of $(a+b)^n$, where n is a positive integer, in the expansion where $(a+b)^n$ is multiplied with a linear or quadratic expression. Use arithmetic and geometric progressions to solve problems in more complex cases.

- Identify the domain and the range of a given function in more complex cases, i.e. with double fractions. Determine whether or not a given function is one-one. Find the domain for which a given function will be one-one. Sketch graphs of functions and their inverses for a specified domain. Form composite functions in more complex cases, and recognise when a given function can be expressed as a composite.

- Apply the relationship between quadrilaterals and triangles with their associated algebraic equations if diagrams are given. Interpret the relationship between algebraic equations to calculate unknown coordinates.

- Solve complex problems concerning the arc length, sector area of a circle and segment area in complex combined figures.

- Apply the trigonometric identities to prove identities and solve trigonometric equations in more complex cases. Find solutions, in degrees or radians and within a specified interval, of the equations $\sin(nx) = k,\ \cos(nx) = k,\ \tan(nx) = k,$ including equations easily reducible to these forms, where n is a small integer or a simple fraction. Solve complex problems concerning the arc length, sector area of a circle and segment area in combined figures. Find the amplitude and period and interpret graphs of the form $y = a\sin(bx) + c,\ y = a\cos(bx) + c,$ and $y = a\tan(bx) + c$.

- Apply the knowledge of the magnitude of a vector and the scalar product of two vectors. Use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors. Apply the knowledge of vectors in three dimensional diagrams.

- Apply the knowledge of the derivative of x^n (for any rational n), together with constant multiples, sums, differences of functions, and of composite functions using the chain rule. Apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change to solve contextual problems involving displacement, velocity and acceleration in simple cases. Locate stationary points, and distinguish between maximum and minimum points and points of inflexion.

- Use the knowledge of integration as the reverse process of differentiation to integrate $(ax + b)^n$ (for any rational n except –1), $e^{ax + b}$, $\frac{1}{ax+b}$, $\sin(ax + b)$, $\cos(ax + b)$ and $\sec^2(ax + b)$ together with constant multiples, sums and differences. Apply trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as $\cos^2 x$.

- Solve complex problems involving the evaluation of a constant of integration. Apply definite integration to find the area of a region bounded by a curve and a line or between two curves, the volume of revolution about one of the axes, displacement, velocity and acceleration in more complex cases. Determine whether the trapezium rule gives an over-estimate or under-estimate.

- Apply the remainder and the factor theorem to polynomials and use the factor theorem to solve equations in more complex cases i.e. a combination of the remainder- and factor theorem. Interpret and use the notation $|x|$, and use the relations such as $|a| = |b| = \Leftrightarrow a^2 = b^2$ and $|x-a| < b \Leftrightarrow a-b < x < a+b$ in solving quadratic inequalities. Sketch graphs of $y = a|x+b| + c$. Divide a polynomial of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero).

- Use the laws of logarithms to solve inequalities in one unknown. Use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or y-intercept. Sketch graphs of $y = e^{ax + b} + c$ and $y = \ln(ax + b) + c$.

- Apply the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and apply properties and graphs of all six trigonometric functions for angles of any magnitude. Apply trigonometrical identities for the simplification and exact evaluation of more complex expressions and in the course of establishing and solving quadratic equations. Select an identity or identities appropriate to the context.

- Apply the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites. Differentiate products and quotients. Use the first derivative of a function which is defined parametrically or implicitly.

- Locate, using an explanation, an approximate root of an equation, by means of graphical considerations and/or searching for a sign change. Rearrange a given equation to form an iteration in more complex cases.

*At **Grade c** the learner is expected to:*

- Interpret the complete square form of a quadratic function to determine the turning point of a graph of a quadratic function and sketch the graph. Find the discriminant of a quadratic polynomial and interpret the nature and the number of real roots of the related equation. Find the equation of a graph given sufficient information. Interpret the difference between identities and equations and use identities to determine unknown coefficient in polynomials. Solve quadratic inequalities in one unknown.

- Use the expansion of $(a+b)^n$, where n is a positive integer for more complex cases. Use arithmetic and geometric progressions in simple cases. Use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression. Interpret and apply the Sigma notation in more complex situations.

- Identify the domain and the range of a given function in simple cases. Determine whether or not a given function is one-one. Form composite functions, and recognise when a given function can be expressed as a composite. Interpret and sketch the graphs of the functions. Find the inverse of a one-one function in more complex cases i.e. with fractions.

- Use the relationship between points of intersection of graphs and solutions of equations including, in simple cases, the correspondence between a line being a tangent to a curve and a repeated root of an equation.

- Solve simple problems concerning the arc length, sector area of a circle and segment area.

- Sketch graphs of the form $y = a\sin(bx)+c$, $y = a\cos(bx)+c$, and $y = a\tan(bx)+c$ (for angles of any size, and using either degrees or radians). Use the exact values of the sine, cosine and tangent of $30°\left(\frac{1}{6}\pi\right)$, $45°\ \left(\frac{1}{4}\pi\right)$, $60°\left(\frac{1}{3}\pi\right)$, and related angles. Prove identities and solve trigonometric equations in simple cases.

- Perform addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms. Calculate unit vectors, displacement vectors and position vectors to calculate the angle between two vectors.

- Use the derivative of x^n (for any rational n), together with constant multiples, sums, differences of functions, and of composite functions using the chain rule. Use differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change to solve problems involving displacement, velocity and acceleration in simple cases. Locate stationary points, and distinguish between maximum and minimum points and points of inflexion.

- Use the knowledge of integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except -1), together with constant multiples, sums and differences. Solve simple problems involving the evaluation of a constant of integration.

Apply definite integration to find the area of a region bounded by a curve and a line or between two curves, the volume of revolution about one of the axes, displacement, velocity and acceleration in simple cases.

- Apply the remainder and the factor theorems to polynomials and use the factor theorem to solve polynomial equations or evaluate unknown coefficients. Use the notation $|x|$, and use the relations such as $|a| = |b| = \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ in solving linear inequalities and equations. Sketch graphs of $y = a|x + b|$ or $y = a|x| + b$

- Interpret the relationship between logarithms and indices, and use the laws of logarithms to solve equations. Interpret the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs. Sketch graphs of $y = e^{ax + b}$ and $y = \ln(ax)$.

- Use properties and graphs of all six trigonometric functions for angles of any magnitude. Use trigonometrical identities for the simplification and exact evaluation of expressions. Apply $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$ to solve equations after solving for α and R.

- Use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites. Differentiate products and quotients of more complex expressions.

- Use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as $\cos^2 x$.

- Apply further the idea of 'reverse differentiation' to include the integration of $e^{ax + b}$, $\frac{1}{ax+b}$, $\sin(ax + b)$, $\cos(ax + b)$ and $\sec^2(ax + b)$ (knowledge of the general method of integration by substitution is not required). Use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as $\cos^2 x$. Use sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or under-estimate.

- Locate an approximate root of an equation, by means of graphical considerations and/or searching for a sign change. Interpret the idea of a sequence of approximations which converges to a root of an equation and use its notation. Interpret how a given simple iterative formula of the form $x_n + 1 = F(x_n)$ relates to the equation being solved.

*At **Grade e** the learner is expected to:*

- Complete the square for a quadratic function and use the solution to determine the turning point of a graph of a quadratic function and sketch the graph. Find the discriminant of a quadratic polynomial.

- Use the expansion of $(a+b)^n$, where n is a positive integer for simple cases. Recognise arithmetic and geometric progressions. Use the formulae for the nth term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions. Apply the Sigma notation.

- Use the terms function, domain, range (image set), one-one function, inverse function and composition of functions. Find the inverse of a one-one function in simple cases.

- Interpret the relationship between a graph and its associated algebraic equation. Use the relationship between points of intersection of graphs and solutions of equations.

- Interpret the term radian and use the relationship between radians and degrees.

- Use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations. Sketch graphs of basic trigonometric graphs.

- Use rectangular Cartesian coordinates to locate points in three dimensions, and use standard notations for vectors. Use unit vectors, displacement vectors and position vectors.

- Interpret the idea of the gradient of a curve, and use the notations of first and second derivative. Use displacement, velocity and acceleration in simple cases.

- Use the knowledge of integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except -1). Solve simple problems involving the evaluation of a constant of integration. Use the knowledge of integration in displacement, velocity and acceleration in simple cases.

- Use the notation of absolute value and solve equations of absolute value. Use the remainder and factor theorem to solve linear equations in one unknown.

- Use the laws of logarithms (excluding change of base). Use logarithms to solve simple equations of the form $a^x = b$. Use e^x and $\ln x$ and their graphs.

- Recognise the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent. Rewrite $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$ to solve for α and R.

- Use the derivatives of e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites in simple cases.

- Use further the idea of 'reverse differentiation' to include the integration of e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax + b)$, $\cos(ax + b)$ and $\sec^2(ax + b)$. Use the trapezium rule to estimate the value of a definite integral in simple cases, or where the diagram is given.

- Use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.

9. LIST OF MATHEMATICAL FORMULAE, NOTATIONS AND OTHER RELEVANT INFORMATION (MF9)

PURE MATHEMATICS

Algebra

For the quadratic equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

For an arithmetic series:

$$u_n = a + (n - l)d, S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - l)d\}$$

For a geometric series:

$$u_n = ar^{n-1}, \qquad S_n = \frac{a(1-r^n)}{1-r}\ (r \neq 1), \qquad S_\infty = \frac{a}{1-r}\ (|r| < 1),$$

Binomial expansion:

$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3} + \ldots + b^n$, where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \ldots$, where n is a rational and $|x| < 1$

Trigonometry

Arc length of circle = $r\theta$ (θ in radians)

Area of a sector of a circle = $\frac{1}{2}r^2\theta$ (θ in radians)

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

$$\cos^2\theta + \sin^2\theta \equiv 1. \qquad 1 + \tan^2\theta \equiv \sec^2\theta, \qquad \cot^2\theta + 1 \equiv \operatorname{cosec}^2\theta$$

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2\sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Principal values: $-\frac{1}{2}\pi \leq \sin^{-1} x \leq \frac{1}{2}\pi$; $0 \leq \cos^{-1} x \leq \pi$; $-\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$

Differentiation

f(*x*)	**f′(*x*)**
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec } x$	$-\text{cosec } x \cot x$
$\cot x$	$-\text{cosec}^2 x$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
uv	$u\frac{\mathrm{d}v}{\mathrm{d}x}+v\frac{\mathrm{d}u}{\mathrm{d}x}$
$\frac{u}{v}$	$\frac{v\frac{\mathrm{d}u}{\mathrm{d}x}-u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

If $x = \mathrm{f}(t)$ and $y = \mathrm{g}(t)$ then $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$

Integration

f(*x*)	$\int \mathbf{f}(x)\mathbf{d}x$	
x^n	$\frac{x^{n+1}}{n+1}+c$	$(n \neq -1)$
$\frac{1}{x}$	$\ln \lvert x \rvert + c$	
e^x	$e^x + c$	
$\sin x$	$-\cos x + c$	
$\cos x$	$\sin x + c$	
$\sec^2 x$	$\tan x + c$	
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$	
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a+x}\right\rvert$	

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = uv - \int vu\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

Vectors

If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$$\mathbf{a.b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$$

Numerical integration

Trapezium rule:

$$\int_a^b f(x)dx \approx \frac{1}{2}h\{y_0 + 2(y_1 + y_2 + ... + y_{n-1}) + y_n\}, \text{ where } h = \frac{b-a}{n}$$

The following list summarises the notation useful to NSSCAS level Mathematics content. Although not all of these notations are used, some of the notations are useful.

Set Notations

$\in$	"…is an element of…"
$\notin$	"…is not an element…"
$\{x : \ldots\}$	the set of x such that ….
n(A)	the number of elements in set A
$\mathbb{N}$	set of natural numbers, $\{1,2,3,\ldots\}$
$\mathbb{Z}$	set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}'$	the set of irrational numbers
$\mathbb{R}$	the set of real numbers
$\cup$	union
$\cap$	intersection

Miscellaneous symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is strictly less than
$\leq$	is less than or equal to
$>$	is strictly greater than
$\geq$	is greater than or equal to
$<; \ll$	is less than, is much less than
$\leq, \ngtr$	is less than or equal to, is not greater than
$>; \gg$	is greater than, is much greater than
$\geq, \nless$	is greater than or equal to, is not less than
∞	infinity

Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$a : b$	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots. + a_n$
$\sqrt{a}$	the positive square root of the real number a
$\lvert a \rvert$	the modulus of the real number a
$n!$	n factorial for $n \in \mathbb{N}$ $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $r \in \mathbb{N}$, $0 \leq r \leq n$ $\frac{n(n-1)...(n-r+1)}{r!}$, for $n \in \mathbb{Q}$, $r \in \mathbb{N}$

Functions

f	function f
f (x)	the value of the function f at x
f : A $\rightarrow$ B	f is a function under which each element of set A has an image in set B
f : $x \rightarrow y$	the function f maps the element x to the element y
f^{-1}	the inverse of the function f
g o f, gf	the composite function of f and g which is defined by (g o f) or gf (x) = g(f(x))
$\lim\limits_{x \to a} f(x)$	the limit of f(x) as x tends to a
Δx; δx	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the nth derivative of y with respect to x
$f'(x), f''(x), \ldots, f^{(n)}(x)$	the first, second and nth derivatives of f(x) with respect to x
$\int y\,dx$	indefinite integral of y with respect to x
$\int_a^b y\,dx$	the definite integral of y with respect to x for values of x between a and b
$\frac{\partial y}{\partial x}$	the partial derivative of y with respect to x
$\dot{x}, \ddot{x}, \ldots$	the first, second, …. derivatives of x with respect to time

Exponential and logarithmic functions

e	base of natural logarithm
e^x, exp x	exponential function of x
$\log_a x$	logarithm to the base a of x
ln x, $\log_e x$	natural logarithm of x
lg x, $\log_{10} x$	logarithm of x to the base 10

Circular and hyperbolic functions

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, $\operatorname{cosec}^{-1}$, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions

Vectors

a	the vector **a**
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment *AB*
$\hat{\mathbf{a}}$	a unit vector in the direction of **a**
i, j, k	unit vector in the direction of the Cartesian coordinate axes
$\lvert\mathbf{a}\rvert$	the magnitude of **a**
$\lvert\overrightarrow{AB}\rvert$	the magnitude of $\overrightarrow{AB}$
a . b	the scalar product of **a** and **b**

Test Record Form for NSSCAS Mathematics: Term 1, 2, and 3

	Test Marks								Ave Total Marks
	Term 1			Term 2			Term 3		
	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8	
Maximum Test Mark									
Name									

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